

# Calculating the Surface of a Flat Figure—application of the Definite Integral in the GeoGebra Program Package

Dragana D. Nedić\*, Gordana Jotanović\*, Aleksandar Kršić \*\* and Tijana Paunović\*\*

\*The University of East Sarajevo, Faculty of Transport and Traffic Engineering Doboj, Bosnia and Herzegovina

\*\* College of Business and Technical Education in Doboj, Bosnia and Herzegovina  
e-mail dragana.nedic@sf.ues.rs.ba

**Abstract** – The possibility to activate students' motivation as well as their ability to work independently is the greatest advantage of using computers in education. The topic of this work is using computers in teaching mathematics shown in the example of the lesson „ Calculating the surface of a flat figure – the application of definite integral. The research was conducted on first-year students of Traffic Engineering. The purpose of this research was to master the same school program in two different methods: the first was the traditional method and the second was using computers, or more precisely, the GeoGebra software. The aim of the testing conducted after teaching the above-mentioned unit was to show the effect of using computers in teaching, and the survey conducted among the students aimed to show their opinion about, and attitude towards, using computers in education.

**Key words:** definite integral, GeoGebra, surface, multimedia learning.

## I. INTRODUCTION

Integral calculus is studied at all technical faculties within the subject of mathematics. The term 'definite integral' can be difficult to understand, which especially refers to students with poor or no previous knowledge of integral calculus. To bring the students closer to the term definite integral and to help them learn this term, as well as other necessary terms easier, we used the GeoGebra mathematics teaching software. The efficient use of new teaching techniques can be very productive for students' progress. Multimedia can be used to encourage students to learn. It is easier to memorize new information if a visual image is present. Baddeley and Hitch [1] have suggested a theory of access memory which consists of two components: visual and verbal. In this way, it was made possible for students to process the information using both eyes and ears. This method of learning approach can be very useful. Pavilio has also suggested that information presented both visually and verbally is easier to memorize, which was applied by Mayer [2,3].

Tall [4] believes that it is very important to link images with certain conditions so that students could obtain additional knowledge. It is necessary to combine images and methods of defining the matter to improve the existing knowledge. Today, the multimedia approach is greatly involved in education. Some of the papers in which we encounter such an approach to work are [5-10]. It is very useful in teaching mathematics, explaining mathematical ideas, abstract terms, theorems, problems... The purpose of this research is to establish to what extent the application of modern technology can help make progress in learning, and to what extent students are interested in using new technology.

## II. CALCULATION OF THE DEFINITE INTEGRAL USING THE GEOGEBRA SOFTWARE

The definite integral was processed using mathematical software GeoGebra, which was used for calculus and visualization. The software presented pictures and graphics to the students. To use computers in teaching mathematics, we can follow the suggestions given in [4] that can be followed.

The definite integral is a fundamental concept in mathematics and is applied not only in mathematics but also in mechanics, physics, and technics. The problems of the calculating path, workforce, and moment of inertia are reduced to the concept of the definite integral, i.e. integral sum and its limit value.

## III. METHOD

The research was conducted at the Faculty of Traffic Engineering in Doboj, three school years.

The research aimed to master the same program in two different ways and, after the testing, to compare the achieved results.

The students were divided into two groups of 20 members. The groups were balanced in terms of prior knowledge.

The first group was taught interactively using computer software, where the computer was used to present pictures, graphs, and interactive illustrations.

After the lectures, the students did a survey whose purpose was to determine the interest of students for this way of teaching.

The results of the test showed that the results achieved by the groups with which the computer was used in the teaching process were better achieved.

The results of the survey conducted by 60 students (the groups with which the computer was used in the processing of teaching content) give priority to using computers.

#### IV. RESULTS AND DISCUSSION

Results of software solving of the problem of defining and calculating a curvilinear trapezoid.

Curvilinear trapezoid in the plane  $xOy$  is called a figure that is constrained by continuous, non-negative function  $y = f(x)$ ,  $x$ -axis and lines  $x = a, x = b$  (Figure 1.) Using the GeoGebra software, the curvilinear trapezoid, the upper sum, and the lower sum are displayed.

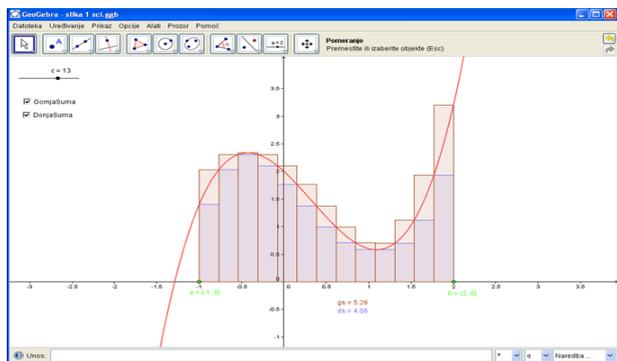


Figure 1. Curvilinear trapezium

#### A. Application of the definite integral-The surface of a flat figure

Let  $f$   $g$  and be continuous functions on  $[a,b]$  and let it be  $f(x) \geq g(x)$  for each  $x \in [a,b]$ , then the surface of the plane figure is curved  $y = f(x)$ ,

$y = g(x)$  and lines  $x = a, x = b$  define as

$$P = \int_a^b (f(x) - g(x)) dx \quad (1)$$

The illustration is in Figure 2.

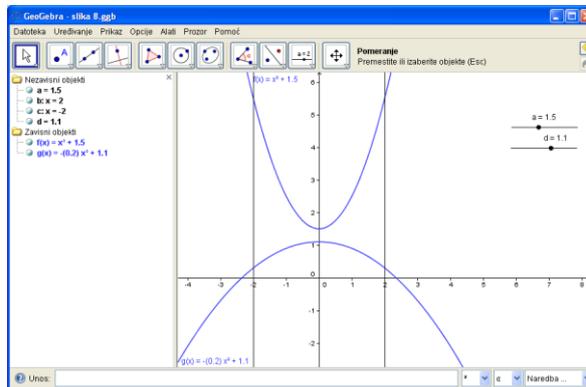


Figure 2. The surface of a flat figure is curved  $y = f(x)$ ,  $y = g(x)$ , lines  $x = a$  and  $x = b$

#### B. The examples of animation in the GeoGebra program

After the processed theoretical part using software, ie applets on which the contents are mentioned and presented, a series of examples follows, some of which are given here.

In the following example, the observed area is bounded by the curve  $y = x^4 - 5x^2 + 4$  and  $Ox$ -axis.

Using the slider, we play an animation where the given function can be clearly seen (Fig.3.), then for example the upper sum (Fig. 4.), or the observed area (Fig. 5.) and finally we have the calculated area of the observed surface (Fig. 6).

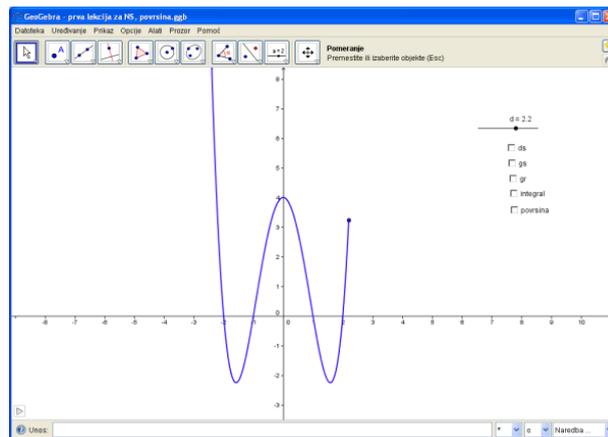


Figure 3. Function  $y = x^4 - 5x^2 + 4$

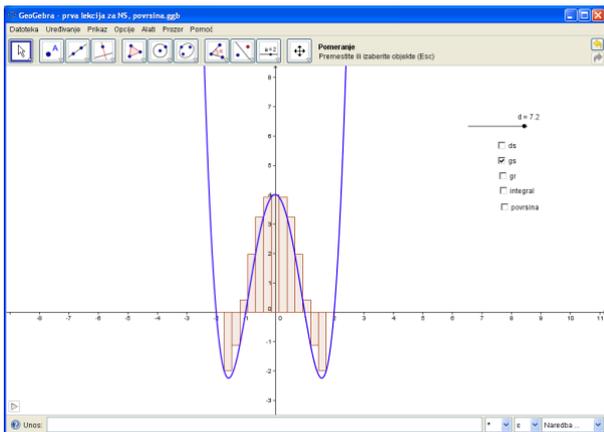


Figure 4. Upper sum

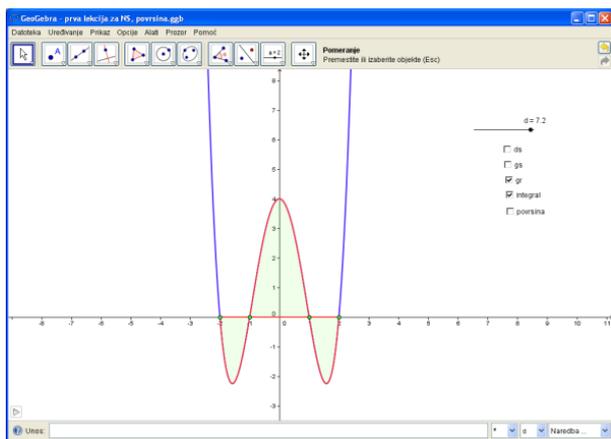


Figure 5. Area

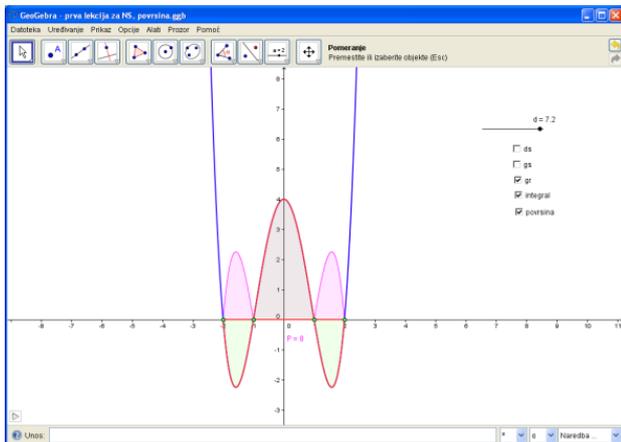


Figure 6. The surface is limited by the curve  $y = x^4 - 5x^2 + 4$  and  $Ox$  - axis

GeoGebra software allows us to change the function using the slider. The table is, say, quite good for working in groups. In Figures 7. and 8. it is clear that by moving the slider we obtained two different surfaces, in Fig.7. we observe the area bounded by the curve  $y = x^2 - 1$ , osom  $Ox$ ,

pravim  $x=1$  and  $x=2$ , while in the image, the value of the parameter was changed using the slider, and we obtained a new function and thus a new observed surface. GeoGebra software allows you to see and set more examples in a shorter time, and if students are given access to a computer or if they work in a computer room, they can explore on their own.

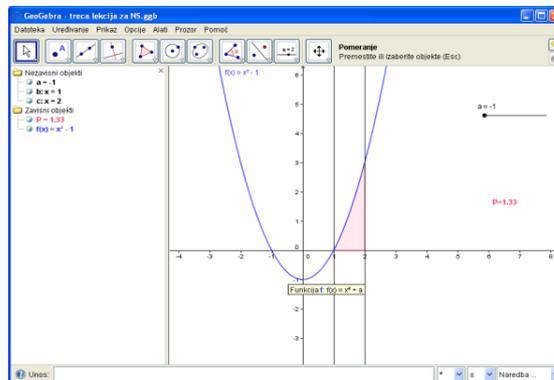


Figure 7. The surface is limited by the curve  $y = x^2 + a$ , for  $a = 1$ ,  $Ox$  - axis, lines  $x = 1$  and  $x = 2$

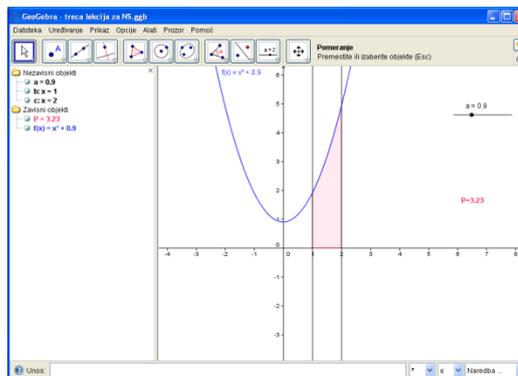


Figure 8. The surface is limited by the curve  $y = x^2 + a$ ,  $Ox$  - axis, lines  $x = 1$  and  $x = 2$

#### IV. CONCLUSION

The results of the research suggest the necessity to introduce changes in the education system and to adjust to the new, modern standards of teaching.

The multimedia approach is successfully used in the creative process, making decisions, solving problems, active participation of students in the class, while the traditional approach is based on the passive position of students concentrated merely on memorizing information.

Modern mathematics teaching relies on the use of computers and educational software. The

introduction of educational software in teaching mathematics has made traditional teaching more modern in terms of speed, quality, and quantity of tasks performed.

Mathematics software package GeoGebra enables a more creative approach to mathematics teaching and better visualization of terms introduced. With the help of mathematical educational software, the students have to adapt to the new circumstances, in which they build mathematical concepts and rules with the help of teachers, but also reveal the required properties independently.

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