Mathematical Problems in Final Exams Through the PISA Perspective

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Abstract – Assessing student's knowledge is a challenge of global interest for general prosperity. Its outcomes are primarily used for educational strategy development. A particular PISA assessment program will be presented. The PISA principles will be employed in the analysis of the mathematical problems in final exams intended for the end of primary/compulsory educational cycle in Serbia.

I. INTRODUCTION

In many occasions we are proud for the achievements of Serbian students in various international scientific competitions [14-22]. In the same time, experiences from the immediate environment indicate Mathematics as an unpopular subject in Serbian educational system [23]. Most people have forgotten ordinary mathematical facts useful in daily routine. This means that gained mathematical knowledge is not functional.

The problem is not local. The surrounding countries, and the others all over the word face the similar situation [3,6,24]. Mathematical content knowledge is not enough for solving real problems and people rely on ever-available technology too often.

The Organization for Economic Co-operation and Development (OECD) is an international organization that aims to increase prosperity, equality, opportunity and well-being for all. Global progress is closely related with capabilities as products of education, hence the OECD Program for International Student Assessment (PISA) has been founded in 1997. The primary purpose of the PISA is systematically monitoring quality adequacy of education [1,6]. PISA outcomes could be benefit for government and should guideline educational strategy.

By the decision of the government, Serbia is included in PISA testing since 2001 [5]. PISA achievements of Serbian students are significantly below the average PISA achievements [5,7]. According to PISA 2012 data, mathematical literacy of almost 40% students in Serbia is not functional, i.e. their achievements are at the first or below the first level of knowledge. Less than 5% of students reach the highest achievements in mathematics, which indicates that our educational system does not provide enough incentives to students with high potential [2].

PISA framework is usually employed in assessing student's knowledge, and quality and equity of education. Having Mathematical literacy in focus, this article considers mathematical problems in the PISA tests, thoughtfully designed to reflect the goals. Each problem has specific mathematical content and context, demands particular process in solving procedure and reflects the level of student's capabilities. PISA framework implies various types of classifications for mathematical problems.

PISA classification/methodology will be used for analyzing mathematical problems in final exams at the end of compulsory education in Serbia. PISA test and final exam are comparable, because both are intended to students of the age 15 for validation of their mathematical knowledge.

II. ON PISA PROGRAM

Throughout various questionnaires intended to students, school principals, teachers and parents, PISA systematically monitors trends in the student's knowledge and skills, and entire social perspective influencing educational process. PISA determines the influence of various social factors on the students' educational achievements (features of the education system, family environment, characteristics of the school and teachers, traits of students…) [5]. The PISA assessment provides three main types of outcomes [1]:

1. Basic indicators on the knowledge and skills of students
2. Indicators that show how the skills relate to various demographic, social, economic and educational variables

3. Indicators on trends that show changes in outcomes and their distributions, and in relationships between student-, school- and system-level background variables and outcomes.

The first outcome reflects educational quality, while the other two indicates two dimensions of equity: inclusion (ensuring that all students attain essential foundational skills) and fairness (distribution of opportunities to acquire a quality education and, more specifically, to the degree to which background circumstances are related to students' education outcomes) [6].

Major domains for testing student's capabilities are reading, mathematics and science. Within each of the domains, questions/problems are particularly chosen to reflect three aspects of literacy and hence, the specific framework is created.

III. MATHEMATICAL LITERACY

Elementary mathematical knowledge is actually the mathematical content knowledge i.e. problem-solving skill [8,10]. In the teaching process focus is on employing the knowledge of arithmetic, algebra, geometry and data for solving as much wide as possible range of mathematical problems (usually posted in a predicted form).

Mathematical literacy is defined by OECD, but the definition is generally accepted. Mathematical literacy is as an individual's capacity to identify and understand the role that mathematics plays in the world, to make well-founded judgments and to use and engage with mathematics in ways that meet the needs of that individual's life as a constructive, concerned and reflective citizen. Mathematical literacy is concerned with the ability of students to analyze, reason, and communicate ideas effectively as they pose, formulate, solve, and interpret solutions to mathematical problems in a variety of situations [1,6].

Mathematical literacy is an important intermediate stage towards the mathematical maturity that imposes the quality of general understanding and operating mathematics. Mathematical maturity is not rigorously defined, but it reflects ability to do the following [9]:

- Make and use connections with other problems and other disciplines
- Fill in missing details
- Spot, correct and learn from mistakes
- Winnow the chaff from the wheat, get to the crux, identify intent
- Recognize and appreciate elegance
- Think abstractly
- Read, write and critique formal proofs
- Draw a line between what you know and what you don't know
- Recognize patterns, themes, currents and eddies
- Apply what you know in creative ways
- Approximate appropriately
- Teach yourself
- Generalize
- Remain focused
- Bring instinct and intuition to bear when needed.

Student's achievements in content knowledge and maturity are mutually independent [10]. Mathematical literacy, with all its achievements' levels, is major conjunction between them showing slow but permanent student's progress.

A. Mathematics in Serbian educational system

Aims and objectives of teaching mathematics in Serbia are declared by government, Ministry of Education, Science and Technological Development. Students' achievements are also determined by the government in the rule book on standards of achievement [4]. Educational standards are expressed throughout general capabilities: elementary, middle and advanced. Each of them is precisely described by adverbs explaining what student can do in each of the five mathematical content category: **N**umbers and operations with them, **A**lgebra and functions, **G**eometry, **M**easurement and **D**ata Processing. Specific mathematical capabilities are defined separately for three various mathematical domains: **M**athematical knowledge and reasoning, **A**pplication of mathematical knowledge and skills to problem solving and **M**athematical communication [11].

Definition of educational goals and learning outcomes is based on Bloom's taxonomy according to which achievement levels are divided into three areas: cognitive, psychomotor and afective [12]. Cognitive and psychomotor achievements are closely related with mathematical content knowledge. The areas are systematized hierarchically and levels of each category contain key verbs which defines qualitative and quantitative learning outcomes.

B. PISA mathematical framework

In the domain of mathematics the three aspects of literacy, present in the posted problems, are the following:

- Content – numbers, algebra and geometry, as familiar curricular subjects, are mixed and overlapped, and settled in one of the four content categories: **Q**uantity, **C**hange and relationships, **S**pace and shapes, **U**ncertainty and data.
- Context – it determines an environment in which the problem is placed; and it can be: **P**ersonal, **O**ccupational, **So**cietal and **Sc**ientific.
- Processes – they describe what students do in problem solving. There are three categories: **F**ormulating situations mathematically; **E**mploying mathematical concepts, facts, procedures and reasoning; and **I**nterpreting, applying and evaluating mathematical outcomes.

The processes are draw on seven fundamental mathematical capabilities (explained in [1].):

1. communicating;
2. mathematising;
3. representing;
4. reasoning and arguing;
5. devising strategies for solving problems;
6. using symbolic, formal and technical language and operations; and
7. using mathematical tools.

The capabilities related with the processes are crucial in defining what it means to be at different levels of the mathematical literacy scales. The six levels of the scale are described through student's ability of doing [1, Chapter 3.]. The first level is the lowest one, and the sixth one is the best. Students are categorized within the levels according to the questionnaire score.

The levels are comparable with the achievement's levels in Serbian educational nomenclature, but more precise.

PISA recommends each mathematical questionnaire to have optimal attendance/distribution of problems within the various aspects:

$$\text{Content: } \begin{pmatrix} Q & C & S & U \\ 25\% & 25\% & 25\% & 25\% \end{pmatrix}$$

$$\text{Context: } \begin{pmatrix} P & O & So & Sc \\ 25\% & 25\% & 25\% & 25\% \end{pmatrix}$$

$$\text{Processes: } \begin{pmatrix} F & E & I \\ 25\% & 50\% & 25\% \end{pmatrix}.$$

IV. PISA PROGRAM AND MATHEMATICS IN SERBIAN EDUCATIONAL SYSTEM

In Serbia, primary school graduates need to take a final exam, which is consisted of three parts. Mathematics makes one of them. The mathematics test is consisted of 20 problems.

The aim of the research proposed here is to analyze structure of mathematics final exam tests according to three aspects of literacy which PISA recommended.

Research covers the sample of 60 mathematical problems, from school years 2016/2017, 2017/2018. and 2018/2019.

The research results are presented below.

C. Mathematical content

In all three observed years the problems from the categories of *Change and relationships* and *Space and shape* are noticed as most frequent, more than other two remaining categories in *Mathematical content* aspect. In the category *Change and relationships* the problems that appear the most are the problems such as manipulating algebraic expressions, solving equations and, in general, traditional mathematical content. In the category *Space and shape*, we have observed traditional geometry,calculating areas and volumes, to be most common. However, the other problems, like the problem of symmetry, also occurs. The next table presents the categorical distribution, i.e. percentage of the share of each category in the aspect of *Mathematical content*.

	Q	C	S	U
2016/17.	10%	25%	40%	15%
2017/18.	20%	35%	40%	5%
2018/19.	15%	40%	30%	15%

D. Mathematical context

In the next table we present the share percentage of the categories in the aspect of *Context* in final exams. In all three years observed, the category *Scientific* is the most common. This category can be roughly divided into two separate parts. Namely, the one part represents the problems concerning the application of mathematics in other fields of science

and technology, while the other part presents simply mathematics as such, irrespective of the world around us [1]. In all three tests the problems of the other part are the most frequent. Aside of that, it is noticeable that the problems from the category *Occupational* are barely represented. Among others, this category encompasses work related problems, like accounting, scheduling, design and job-related decision making [1].

	P	O	So	Sc
2016/17.	30%	5%	20%	45%
2017/18.	15%	10%	4%	55%
2018/19.	35%	0%	15%	50%

E. Mathematical process

In the aspect of *Mathematical process* PISA recommends a higher representation of the category *Employing mathematical concepts, facts, procedures and reasoning*, which was met by us, though significantly more than recommended. Problems that occur quite often are the problems concerning manipulating numbers, algebraic expressions, as well as the ones related to the application of mathematical facts, rules and algorithms when finding solutions. All of these activities, among others, define *Employing* category [1]. Table below shows percentage of each category in the aspect of *Mathematical process*.

	F	E	I
2016/17.	5%	80%	15%
2017/18.	25%	65%	10%
2018/19.	10%	75%	15%

V. CONCLUSION

The research confirms significant deviation between aspectual structures of the final exam tests and PISA tests.

According to the mathematical content, the most represented problems are the ones related to the traditional algebraic and geometric concepts. This may cause students to always expect the same patterns in dealing with problems in mathematics.

The context is an aspect where the deviation is the largest. Within the whole sample a half posted problems is settled in scientific context. The most problems have purely abstract formulation: solve the equation, simplify the expression, determine *x*,… It could be hard and tedious for students to even consider the problems. This fact supports low level of applicability of gained mathematical knowledge. On the other hand, simple and short formulations enable recognition of suitable procedure for problem solving and therefore, teachers resort to this method. Absence of real situations related to problems, inhibits the infiltration of mathematics into everyday life.

Context of a problem imposes solving procedure and determines necessary mathematical processes. The simple and short formulations commonly imply single and simple process, which comes down to employing mathematical concepts, facts, procedures and reasoning. Remaining types of mathematical processes cannot come to the fore without compound problem formulation. Hence, domination of scientific context is obviously significantly correlated with large representation of only one type of mathematical process.

Educational system in Serbia mostly supports mathematical content knowledge. Problems and tasks are commonly settled in scientific context, very abstract and far from reality. Students are encouraged for gaining routine within mathematics as a subject. More evident problems in teaching mathematics are emphasized in [13]. Therefore, mathematical maturity and literacy are achievable to a small number of students – to those who have natural predispositions for mathematics.

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